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TEKISLIKDA BO'LAKLI SILLIQ FUNKSIYANING POTENSIALI MAXSUSLIKKA EGA BO'LGAN SHREDINGER OPERATORINING XOS FUNKSIYALARI BO'YICHA FURYE QATORIGA YOYISH

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Annotatsiya. Ushbu ish singulyar potensialli Shredinger operatorining xos funksiyalari bo'yicha berilgan bo'lakli-silliq funksiya Furye qatorining nuqtada yaqinlashishi masalalasiga bag'ishlangan.

Kalit so`zlar: Shredinger operatori, funksiyalar, teoremaning isboti, funksiyalarning spektral yoyilmalari.

Bo'lakli-silliq funksiyalarning spektral yoyilmalarini yaqinlashishga tekshirish masalalalari dolzarb masalalardan hisoblanadi. Bu sohada ko'plab natijalar olingan. Bu sohada hal qilingan asosiy natijalar va mavjud muammolar haqidagi ma'lumotlar [1,2,3] ishlarda keltirilgan. Keltirilgan ishlarda bo'lakli-silliq funksiyalarning silliq koeffitsiyentli ikkinchi tartibli elliptik operatorlar bilan bog'liq spektral yoyilmalarining yaqinlashish masalalari tadqiq qilingan. Operator singulyar potensialli Shredinger bo'lgan holda shu maslalalar [4,5] ishlarda o'rganilgan.

Ω - tekislikdagi markazi koordinata boshida radiusi R ga teng bo'lgan doira, $q(x,y)=q(\sqrt{x^2+y^2})$ esa nomanfiy istalgan radial simmetrik funksiya bo'lsin. $L_2(\Omega)$ fazoda aniqlanish sohasi $D(L)=C_0(\Omega)$ bo'lgan quyidagi

$$Lu(x,y)=-\Delta u(x,y)+q(x,y)u(x,y) \qquad (1)$$

Shredinger operatorini qaraymiz, bu yerda $q(x,y)=q(r)$ potensial

$$\int_0^h rq^2(r)dr < \qquad (2)$$

shartni qanoatlantiruvchi funksiya.

Shuni ta'kidlash joizki, (2) shartni qanoatlantiruvchi potensial nol nuqtada maxsuslikka ega bo'lishi mumkin.

Aytaylik $u(x,y)$ funksiyalar

$$\Delta u(x,y)+\lambda u(x,y)=q(x,y)u(x,y) \qquad (3)$$

Tenglamaning

$$u(x,y)|_{\partial\Omega}=0 \qquad (4)$$

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chegaraviy shartni qanoatlantiruvchi noldan farqli yechimi bo'lsin. Biz bu funksiyalarni xos funksiyalar deb ataymiz. Ma'lumki L operator uchun (4) Dirixle chegaraviy sharti bajarilganda L operator simmetrik bo'ladi va uni K. Fridrixs teoremasiga asosan o'z-o'ziga qo'shma kengaytirish mumkin. Bizning holimizda L operatorning barcha xos sonlari va xos funksiyalari oshkor topiladi. Agar karraliligini ham hisoblasak (1) operatorning xos sonlarini o'sish tartibida $\{0 < \lambda_1 < \lambda_2 \quad \lambda_3 \quad ...\}$, kabi ularga mos xos funksiyalarni esa $u_n(x,y)$ bilan belgilaymiz. Istalgan $f \quad L_2(\Omega)$ funksiyaning $u_n(x,y)$ xos funksiyalar bo'yicha Furye koeffitsiyentini f_n bilan belgilaymiz.

Berilgan f funksiya Furye qatorining qismiy yig'indisi deb

$$E_\lambda f(x,y) = \sum_{\lambda_n < \lambda} f_n u_n(x,y) \qquad (5)$$

yig'indiga aytamiz. Aytaylik G - soha, chegarasi $\partial G = \{(x,y) \quad \Omega : b^2x^2 + a^2y^2 = a^2b^2\}$ ellipsdan iborat qism soha bo'lsin. G sohaning xarakteristik funksiyasini χ_G orqali belgilaylik, ya'ni

$$\chi_G(x,y) = \begin{cases} 1 \ agar \ (x,y) \quad G \\ \frac{1}{2} \ agar \ (x,y) \quad \partial G \\ 0 \ agar \ (x,y) \quad \Omega \setminus \overline{G} \end{cases}$$

Teorema 1. Aytaylik q potensial (2) shartni qanoatlantiruvchi istalgan funksiya bo'lsin. U holda istalgan $(x,y) \quad \Omega$ nuqtada $f(x,y) = \chi_G(x,y)$ bo'lakli-silliq funksiyaning Furye qatori yaqinlashadi:

$$\lim_{\lambda \to} E_\lambda f(x,y) = f(x,y).$$

Teoremaning isboti har bir xos funksiya uchun o'rinli bo'lgan Titchmarshning o'rta qiymat formulasi va Sh. Alimov tomonidan ishlab chiqilgan usulga asoslanadi, bunda L operatorning xos funksiyalarining oshkor ko'rinishidan va $q(r)$ potensialning sferik simmetrikligidan foydalaniladi.

$E_\lambda f(x,y)$ qismiy yig'indining fundamentalligini ko'rsatamiz.

$$\left|E_{\mu+1}f(x) - E_\mu f(x)\right| = \left|\sum_{\mu \ \sqrt{\lambda_n} < \mu+1} f_n(x)u_n(x)\right|$$

$$\left|\sum_{\mu \ \sqrt{\lambda_n} < \mu+1} f_n^2(x)\mu_n^{2\alpha}\right|^{\frac{1}{2}} \left|\sum_{\mu \ \sqrt{\lambda_n} < \mu+1} \frac{u_n^2(x)}{\mu_n^{2\alpha}}\right|^{\frac{1}{2}} \qquad (6)$$

tengsizlik bajarilishini ko'rsatamiz.

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Teorema 2. $\forall x \quad K \quad \Omega$ uchun

$$\sum_{\mu<\mu_n<\mu+1} u_n^2(x) \quad C(K)\mu \tag{7}$$

tengsizlik o'rinli.

Isbot. E.G. Titchmarshning xos funksiyalar uchun o'rta qiymat formulasidan foydalanib isbotlanadi.

Teorema 2 dan quyidagi natijaga ega bo'lamiz.

Natija 1. Teorema 2 ning barcha shartlari bajarilsin. U holda $\forall K \quad \Omega$ uchun har qanday $\alpha > 0$ son uchun

$$\sum_{\mu<\mu_n<\mu+1} \frac{u_n^2(x)}{\mu_n^{2\alpha}} \quad C(K)\mu^{-2\alpha+1} \tag{8}$$

tengsizlik o'rinli.

Isbot.

$$\sum_{\mu<\mu_n<\mu+1} \frac{u_n^2(x)}{\mu_n^{2\alpha}} \quad \mu^{-2\alpha} \sum_{\mu<\mu_n<\mu+1} u_n^2(x) \quad C(K)\mu^{-2\alpha}\mu = C(K)\mu^{-2\alpha+1}$$

(8) bahoni (6) ga olib borib qo'yamiz. Natijada quyidagiga ega bo'lamiz.

$$\left|E_{\mu+1}f(x) - E_{\mu}f(x)\right| \quad \left|\sum_{\mu \sqrt{\lambda_n}<\mu+1} f_n^2(x)\mu_n^{2\alpha}\right|^{\frac{1}{2}} \mu^{\frac{1}{2}-\alpha} \quad C\|f\|_{L_2(\Omega)}\mu^{\frac{1}{2}-\alpha} \tag{9}$$

$\mu^{\frac{1}{2}-\alpha} \to 0$ o'rinli bo'lishi uchun, $\alpha > \frac{1}{2}$ tengsizlik bajarilishi kerak.

$$f_n^2(x) = \left|\left(f(x),u_n(x)\right)\right|^2 = \frac{1}{\mu_n^4}\left|\left(f(x),-\Delta u_n(x) + q(x)u_n(x)\right)\right|^2 =$$

$$\frac{1}{\mu_n^4}\left|\left(f(x),-\Delta u_n(x)\right) + \left(f(x),q(x)u_n(x)\right)\right|^2$$

$$\frac{2}{\mu_n^4}\left|\left(f(x),-\Delta u_n(x)\right)\right|^2 + \frac{2}{\mu_n^4}\left|\left(f(x),q(x)u_n(x)\right)\right|^2$$

$$\sum_{\mu \ \mu_n<\mu+1} \frac{2}{\mu_n^4}\mu_n^{2\alpha}\left|\left(f(x)q(x),u_n(x)\right)\right|^2$$

$$\frac{2}{\mu_n^4}\left(\mu+1\right)^{2\alpha} \sum_{\mu \ \mu_n<\mu+1} \left|\left(f(x)q(x),u_n(x)\right)\right|^2 \quad C\mu^{2\alpha-4}\left\|fq\right\|_{L_2(\Omega)}^2$$

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$\mu^{2\alpha-4} \to 0$ o'rinli bo'lishi uchun, esa $\alpha \quad 2$ tengsizlik bajariladi. Demak, $\frac{1}{2} < \alpha \quad 2$

$$\mu \ \ \mu_n < \mu+1 \frac{1}{\mu_n^4} \mu_n^{2\alpha} \left| (f(x)q(x), u_n(x)) \right|^2 \quad \mu^{2\alpha-4} \quad \mu^{1-2\alpha} = \mu^{-3} \qquad (10)$$

bahoga ega bo'lamiz. ikkinchi tomondan Grin formulasini qo'llab quyidagi

$$\left(f(x), \Delta u_n(x)\right) = \int_S \frac{\partial u_n}{\partial \nu_n} d\sigma$$

tenglikni olamiz.

Lemma 1. $u_n(x)$ funksiya

$$-\Delta u_n(x) + q(x)u_n(x) = \lambda_n u_n(x)$$

tenglamaning noldan farqli yechimi bo'lsin. U holda

$$\int_\theta u_n(x + r\theta)d\theta = 2\pi u_n(x) I_0(r\mu_n)$$

$$+\frac{\pi}{2} \int_0^r W_0(t\mu_n, r\mu_n) t \int_\theta q(x + t\theta) u_n(x + t\theta) d\theta dt$$

formula o'rinli. Bu yerda

$$\int_\theta f(x + r\theta)d\theta = \int_0^{2\pi} f(x_1 + r\cos\theta, x_2 + r\sin\theta)d\theta$$

$I_0(z), Y_0(z)$ - mos ravishda Bessel va Neyman funksiyalari,

$$W_0(a,b) = I_0(a)Y_0(b) - I_0(b)Y_0(a)$$

Isbot. E.G.Titchmarsh kitobida berilgan.

Quyidagi funksiyani qaraymiz.

$$\upsilon(r,\mu) = \frac{1}{2\pi} I_0(r\mu)\eta_R(r),$$

bunda

$$r = |x - y|, \ \mu > 0, \ \eta_R(r) \quad C_0[0;+\ ) \text{ va}$$

$\eta_R(r) \quad 1$ agar $R < r < 2R$ bo'lsa, $\eta_R(r) \quad 0$

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Ushbu funksiyaning $u_n(x)$ funksiyalar bo‘yicha Furye koeffitsiyentlarini hisoblaymiz.

$$\tilde{c}_n(x) = (\upsilon(x), u_n(x)) = \int_{\Omega} \upsilon(|x-y|, \mu) u_n(y) dy$$

$$= \int_{R<|x-y|<2R} \upsilon(|x-y|, \mu) u_n(y) dy = \int_{R}^{2R} r\upsilon(r, \mu) \int_{\theta} u_n(x + r\theta) d\theta dr$$

o‘rta qiymat formulasidan foydalansak, $\tilde{c}_n(x)$ uchun

$$\tilde{c}_n(x) = \int_{R}^{2R} r\upsilon(r, \mu) \left[2\pi u_n(x) I_0(r\mu_n) + \frac{\pi}{2} \int_0^r W_0(t\mu_n, r\mu_n) t \int_{\theta} q(x + t\theta) u_n(x + t\theta) d\theta \right] dtdr$$

$$= \mu \left[u_n(x) \int_{R}^{2R} rI_0(r\mu) I_0(r\mu_n) \eta_R(r) dr \right.$$

$$\left. + \frac{1}{4} \int_{R}^{2R} rI_0(r\mu) \eta_R(r) \int_0^r W_0(t\mu_n, r\mu_n) t \int_{\theta} q(x + t\theta) u_n(x + t\theta) d\theta dtdr \right]$$

ko‘rinishni olamiz. Agar

$$b_n(R, \mu) = \int_{R}^{2R} rI_0(r\mu) I_0(r\mu_n) \eta_R(r) dr$$

$$h_n(R, \mu) = \frac{1}{4} \int_{R}^{2R} rI_0(r\mu) \eta_R(r) \int_0^r W_0(t\mu_n, r\mu_n) t \int_{\theta} q(x + t\theta) u_n(x + t\theta) d\theta dtdr$$

ko‘rinishdagi belgilashlarni kiritsak,

$$\tilde{c}_n(x) = u_n(x) b_n(R, \mu) + h_n(R, \mu) \qquad (11)$$

formulaga ega bo’lamiz.

Lemma 2. $b_n(R, \mu)$ funksiya uchun quyidagi

$$b_n(R, \mu) \quad c_0 \quad R^{-1}, \quad c_0 > 0 \qquad (12)$$

baho o’rinli.

Isbot. Ushbu lemmaning isboti [8] da berilgan.

U holda $\upsilon(r, \mu)$ funksiyaning Furye qatoriga yoyilmasi

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$$\upsilon(r,\mu) = \sum_{n=1} \upsilon_n(y,\mu)u_n(y) \qquad (13)$$

ko‘rinishda bo‘ladi, $r = |y - y|$, $y \quad \Gamma$, $y \quad \Omega$.

Foydalanilgan adabiyotlar

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